GRADIENT BASED RECONSTRUCTIONS FOR ADAPTIVE OPTICS SYSTEMS

Daniela Saxenhuber and Ronny Ramlau

http://eso-ao.indmath.uni-linz.ac.at/ | daniela.saxenhuber@indmath.uni-linz.ac.at

Industrial Mathematics Institute and Johann Radon Institute for Computational and Applied Mathematics, JKU Linz, Austria

Introduction

Large ground-based telescopes rely on Adaptive Optics (AO) systems in order to achieve a good image quality. Due to steadily growing telesope sizes there is a strong increase in the computational load for atmospheric reconstruction with current methods, first and foremost the MVM.

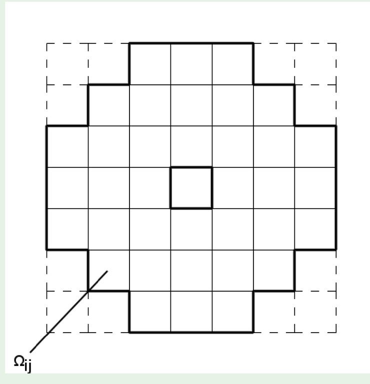
Adaptive Optics system:

Input: Shack-Hartmann wavefront sensor (WFS) data of guide star α_g, $g = 1,\ldots,G$ on each sub-aperture Ω_{ij} of the aperture Ω_D

$\Gamma : H^1(\Omega_D) \to \mathbb{R}^{2\#sub}$, $\#sub$ ... number of active sub-apertures

$$s^x_{\alpha_g} = \frac{1}{|\Omega_{ij}|}\int_{\Omega_{ij}} \frac{\partial \varphi_{\alpha_g}}{\partial x}\, d(x,y)\,, \quad \text{and} \quad s^y_{\alpha_g} = \frac{1}{|\Omega_{ij}|}\int_{\Omega_{ij}} \frac{\partial \varphi_{\alpha_g}}{\partial y}\, d(x,y)\,.$$

Output: DM commands $\mathbf{\Phi}_{DM}$



WFS sub-apertures

Instead of using one big matrix-vector system, one can decouple the problem in 3 steps:

3-step approach[1]:

Solve AO problem sequently:

1. Reconstruct incoming wavefronts from SH WFS data: $\varphi_{\alpha_g} = \mathrm{CuReD}(s^x_{\alpha_g}, s^y_{\alpha_g})$ [2]
2. Atmospheric tomography: Gradient-based method
3. Compute optimal mirror shapes from the reconstructed atmosphere (fitting step): projection of reconstructed atmosphere $\mathbf{\Phi}$ onto DMs

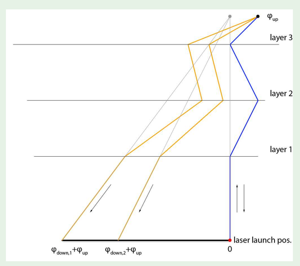
$\Rightarrow$ flexible and fast reconstruction!

AO systems such as Multi Conjugate AO (MCAO), Laser Tomography AO (LTAO) or Multi Object AO (MOAO) all require atmospheric tomography but differ in the projection step. In the following, we propose a gradient based method for the atmospheric tomography. The main goal of this iterative approach is the comparabilty with the MVM method in quality and a considerable reduction of computational cost.

Problem modelling

Tip-tilt indetermination:

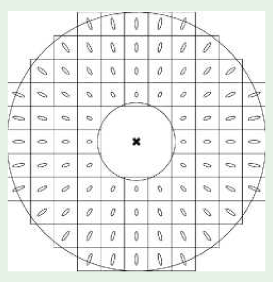
With natural guide stars (NGS) only a low sky coverage reached $\to$ artificial laser guide stars (LGS) are created with laser beacons.
Problem: average of derivatives of incoming wavefronts (tip-tilt) is wrong $\to$ combine LGS and NGS.



SH WFS measurements from LGS are affected by spot elongation due to the non-zero thickness of the sodium layer. Depending on the laser launch position (the height of the LGS and the sodium layer) one can model the corresponding covariance matrix for the noise.

Spot elongation:

For exact WFS data $s_{\alpha_g} = [s^x_{\alpha_g}\ s^y_{\alpha_g}]$ one can model the noisy measurements by: $s^\delta_{\alpha_g} = s_{\alpha_g} + C^{1/2}_{\alpha_g}\eta$, with η white noise.
Thus, $\varphi^\delta_{\alpha_g} = \Gamma^{-1}(s_{\alpha_g} + C^{1/2}_{\alpha_g}\eta)$ and $\mathrm{cov}(\varphi_{\alpha_g}) = \Gamma^{-1}C_{\alpha_g}\Gamma^{-T}$ (with Γ the discretized SH operator). Therefore, $\mathrm{cov}(\varphi) = \Gamma^{-1}C_\eta\Gamma^{-T} =: \overline{C_\eta}$ with $C_\eta = \mathrm{diag}(C_{\alpha_1}, \ldots, C_{\alpha_G})$.



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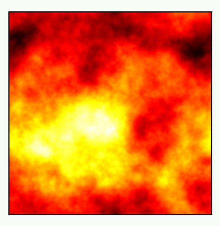
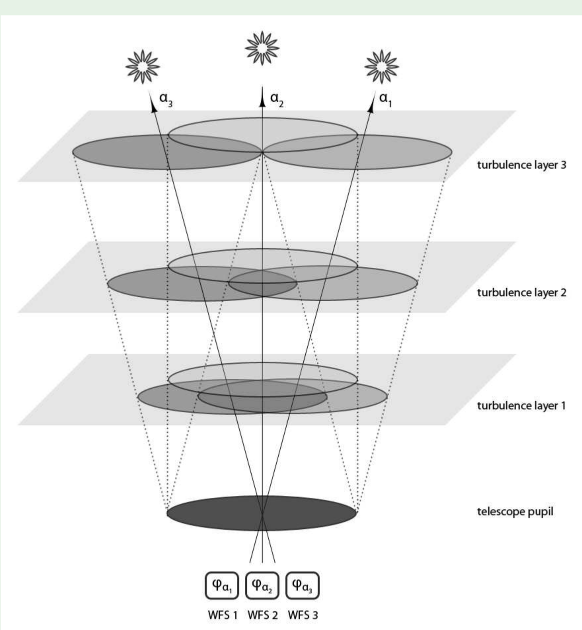
Statistics of the atmosphere:

To model the atmosphere, a finite number layers $l = 1, \ldots L$ is used. Each layer can be described e.g. by the Kolmogorov turbulence model with covariance matrices $C^{(l)}_\Phi$ and the c^2_n-profile γ_l measuring the turbulence strength of layer l.



Atmospheric Tomography

Problem formulation:



Input:

- reconstructed incoming wavefronts φ_{α_g} from LGS $g = 1, \ldots, G$ on $\mathbf{\Omega}_D$ (aperture)
- tip/tilt data $\mathbf{t}_{\beta_n} \in \mathbb{R}^2$ from natural guide stars $n = 1, \ldots, N$

Goal:

- **fast** reconstruction of turbulence layers $\Phi^{(l)}$ on $\mathbf{\Omega}_l$, $l = 1, \ldots, L$

$\Longrightarrow$ **ill-posed inverse problem**, requires **regularization**.

Forward operators:

- LGS data: $\mathbf{A}_{\boldsymbol{\alpha}_g} : \bigotimes_{l=1}^{L} L_2(\mathbf{\Omega}_l) \to L_2(\mathbf{\Omega}_D)$ with geometric light propagation [1]:

$$\mathbf{A}_{\boldsymbol{\alpha}_g}\mathbf{\Phi} := \sum_{l=1}^{L} T^{\alpha_g h_l}\Phi^{(l)} = \varphi_{\alpha_g}(\mathbf{r})\,, \quad \text{with} \quad (T^{\alpha_g h_l}\Phi^{(l)})(\mathbf{r}) := \Phi^{(l)}(c_l\mathbf{r} + h_l\boldsymbol{\alpha}_g)\,,\ \mathbf{r} \in \mathbf{\Omega}_D\,.$$

$$\mathbf{\Phi} = (\Phi^{(1)}, \cdots, \Phi^{(L)})^T\,, \ \ldots \text{ turbulence layers}, \quad c_l := 1 - \frac{h_l}{h_{LGS}}, \quad h_{LGS} \ldots \text{ LGS height.}$$

$$\langle \mathbf{\Phi}, \mathbf{\Psi}\rangle := \sum_{l=1}^{L}\frac{1}{\gamma_l}\langle \Phi^{(l)}, \Psi^{(l)}\rangle_{L_2(\mathbf{\Omega}_l)}\,, \quad \gamma_l \ldots c_n^2\text{-profile of layer } l\ (\sum_{l=1}^{L}\gamma_l = 1)$$

with the L_2-adjoint: $\mathbf{A}^*_{\boldsymbol{\alpha}_g} : L_2(\mathbf{\Omega}_D) \to \bigotimes_{l=1}^{L} L_2(\mathbf{\Omega}_l)$ where

$$\mathbf{A}^*_{\boldsymbol{\alpha}_g}(\mathbf{\Psi}) = (\gamma_1(T^{\alpha_g h_1})^*\mathbf{\Psi}, \ldots, \gamma_L(T^{\alpha_g h_L})^*\mathbf{\Psi})^T,\ (T^{\alpha_g h_l})^*\mathbf{\Psi} = \mathbf{\Psi}(\mathbf{r} - \alpha_g h_l)\chi_{\Omega_D(\alpha_g h_l)}(\mathbf{r})$$

- Tip-tilt data [3]: $\mathbf{N}_{\beta_i} : \bigotimes_{l=1}^{L} H^1(\mathbf{\Omega}_l) \to \mathbb{R}^2$, with

$$\mathbf{N}_{\beta_i}\mathbf{\Phi} = \mathbf{t}_{\beta_i} = \begin{pmatrix} t^x_{\beta_i} \\ t^y_{\beta_i}\end{pmatrix} \in \mathbb{R}^2\,,\ i = 1, \ldots, N, \quad \mathbf{r} = (x,y) \in \mathbf{\Omega}_D \text{ and}$$

$$t^x_{\beta_i} = \sum_{l=1}^{L}\int_{\mathbf{\Omega}_D}\frac{\partial}{\partial x}\Phi^{(l)}(\mathbf{r} + h_l\beta_i)\,d\mathbf{r}\,,\quad t^y_{\beta_i} = \sum_{l=1}^{L}\int_{\mathbf{\Omega}_D}\frac{\partial}{\partial y}\Phi^{(l)}(\mathbf{r} + h_l\beta_i)\,d\mathbf{r}\,.$$

$$\text{Solve } \mathbf{A\Phi} = \varphi \Leftrightarrow \begin{pmatrix}\mathbf{A}_{\alpha_1} \\ \vdots \\ \mathbf{A}_{\alpha_G}\end{pmatrix}\mathbf{\Phi} = \begin{pmatrix}\varphi_{\alpha_1} \\ \vdots \\ \varphi_{\alpha_G}\end{pmatrix} = \varphi \quad \text{with} \quad \mathbf{A}^* = \sum_{g=1}^{G}\mathbf{A}^*_{\boldsymbol{\alpha}_g} + \sum_{n=1}^{N}\mathbf{N}^*_{\beta_n}\,.$$

Least squares solution with penalty term (MAP estimation):

$$J(\mathbf{\Phi}) = \|\mathbf{A\Phi} - \varphi\|^2_{\overline{C_\eta}^{-1}} + \alpha_\Phi\|\mathbf{\Phi}\|^2_{C_\Phi^{-1}} \to \min$$
$$J'(\mathbf{\Phi}) = -2\mathbf{A}^*\overline{C_\eta}^{-1}(\varphi - \mathbf{A\Phi}) + 2\alpha_\Phi C_\phi^{-1}\mathbf{\Phi} =: -\mathbf{d}$$

Steepest descent with stepsize τ:

$$\mathbf{\Phi}_{j+1} = \mathbf{\Phi}_j + \tau_j\mathbf{d}_j$$
$$\tau_j = \min_{t\in\mathbb{R}} J(\mathbf{\Phi}_j + t\mathbf{d}_j)$$
$$= \frac{\sum_{l=1}^{L}\frac{1}{\gamma_l}\langle \mathbf{d}^{(l)}_j, \mathbf{d}^{(l)}_j\rangle_{L^2(\Omega_l)}}{\langle \overline{C_\eta}^{-1}\mathbf{Ad}_j, \mathbf{Ad}_j\rangle_{L^2(\Omega_D)} + \alpha_\Phi\sum_{l=1}^{L}\langle (C^{(l)}_\Phi)^{-1}\mathbf{d}^{(l)}_j, \mathbf{d}^{(l)}_j\rangle_{L^2(\Omega_l)}}$$

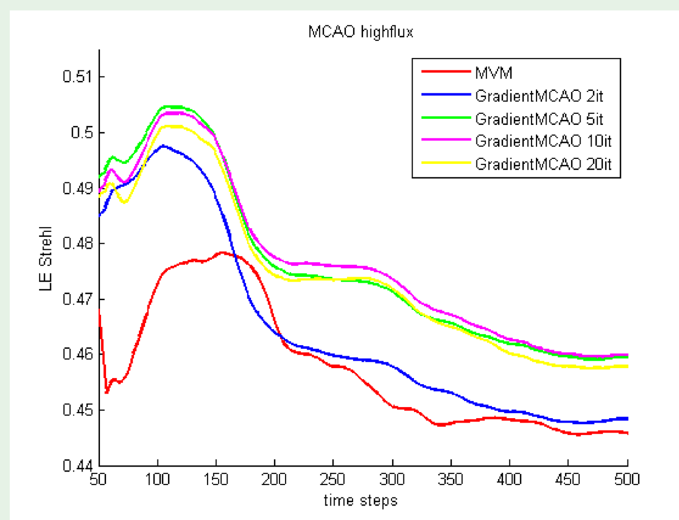
- well parallelizable method
- easily adaptable to changes in asterism
- can also be used without noise statistics ($\overline{C_\eta} = I$, $\alpha_\Phi = 0$) e.g. for highflux, without spot elongation

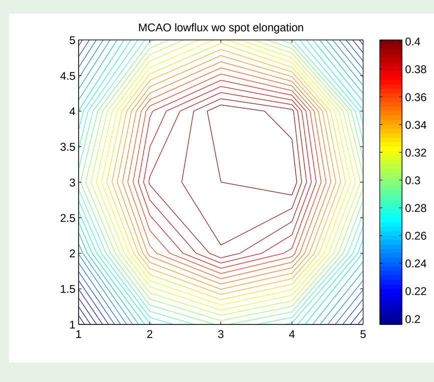
Quality results

All results below were obtained for the E-ELT on the ESO end-to-end simulator, OCTOPUS.

Multi Conjugate AO (MCAO):

- 6 LGS in a circle of radius 1 arcmin
- 3 TTS in a circle of radius 4/3 arcmin
- 3 DMs (0m, 4km, 12.7km)
- 3–layer reconstruction on DMs
- pseudo open loop control (polc)
- wo spot elongation $\to$ no noise models needed
- only few iterations needed





Lowflux (100 photons per sub-aperture)

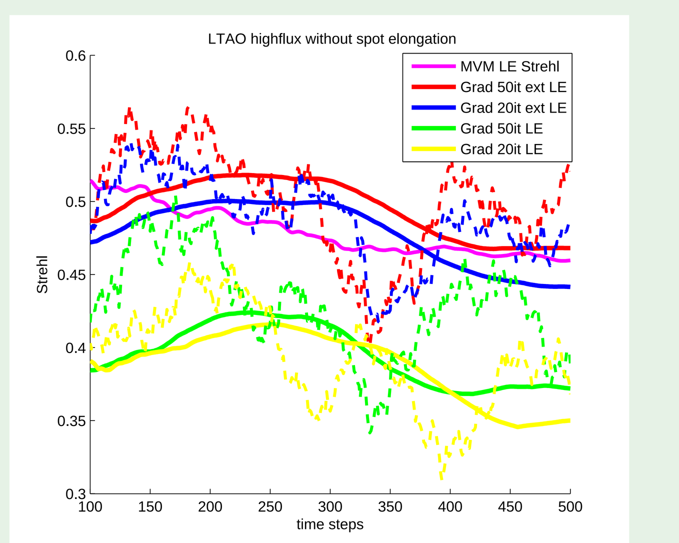
Computational complexity:

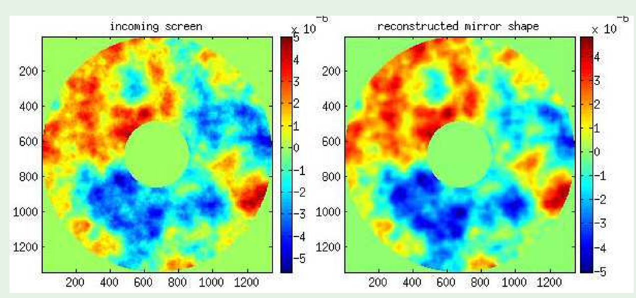
- $n \sim 0.75 \cdot 84^2 = 5292$, with a 84x84 SH-WFS
- CuReD: $20 \cdot n$ per guide star
- Gradient: $\sim ((16L+2)\cdot(G+N)+(L+2))\cdot n$ per iteration, e.g. 3 layers, 6 LGS, 3 NGS: $\sim 455n$
- calculation in guide star directions is parallelizable

Laser Tomography AO (LTAO):

- 6 LGS (3.75 arcmin), 3 NGS (5 arcmin)
- 1 ground DM, polc control
- 9 layer reconstruction followed by projection onto ground DM
- highflux (10000 photons per sub-aperture)
- wo spot elongation $\to$ no noise models needed

	Grad 20it	Grad 50it	MVM
LE center strehl	44.14	46.81	45.96





Multi Object AO (MOAO)

- 6 LGS (3.75 arcmin)
- 3 NGS (5 arcmin)
- 1 ground DM, open loop
- highflux, wo spot elongation

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References

[1] R. Ramlau and M. Rosensteiner: *An efficient solution to the atmospheric turbulence tomography problem using Kaczmarz iteration.* Inverse Problems, 28 (2012)

[2] M. Rosensteiner: *Wavefront reconstruction for extremely large telescopes via CuRe with domain decomposition.* J. Opt. Soc. Am. A, 11 (2012)

[3] R. Ramlau, A. Obereder, M. Rosensteiner, D. Saxenhuber: *Efficient iterative tip/tilt reconstruction for atmospheric tomography*, submitted