A FAST RECONSTRUCTION METHOD FOR COMPLEX ADAPTIVE OPTICS SYSTEMS

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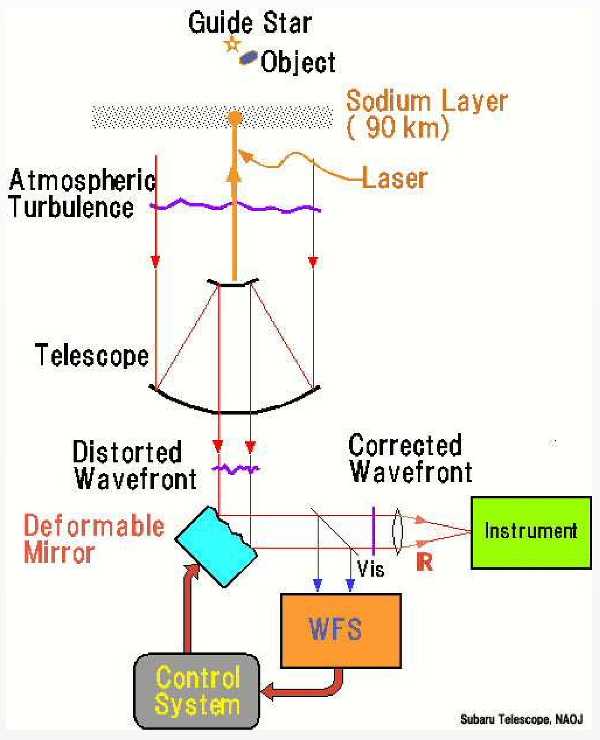
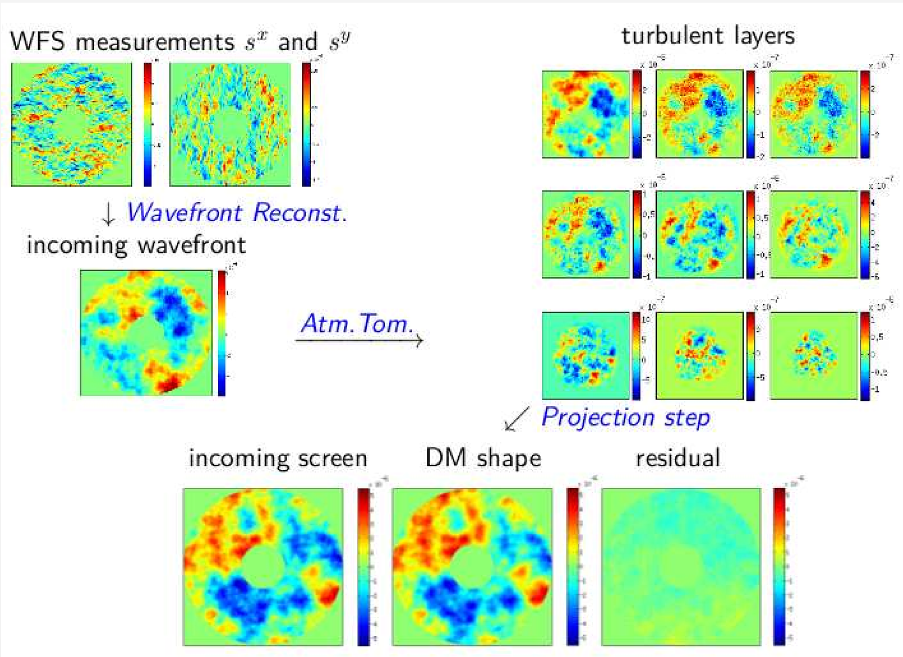
The problem of atmospheric tomography arises in ground-based telescope imaging with Adaptive Optics (AO), where one aims to physically correct atmospheric turbulences via deformable mirrors (DM) in real-time, i.e. at around 500 Hertz. The optimal shape of the DM is determined from wavefront measurements of natural guide stars (NGS) as well as laser guide stars (LGS) and is an ill-posed inverse problem.

Many complex AO systems, such as *Multi-Conjugate Adaptive Optics (MCAO)*, *Laser Tomography Adaptive Optics (LTAO)* or *Multi-Object Adaptive Optics (MOAO)*, depend on a sufficient reconstruction of the turbulence profiles within the required time frame in order to obtain a good correction. Due to steadily growing telescope sizes, there is a strong increase in the computational load for atmospheric reconstruction with current methods, first and foremost the matrix-vector-multiplication (MVM). Instead of using one big matrix-vector system, one can decouple the problem in 3 steps:



3-step approach[1]:
Solve AO problem sequently:

1. Reconstruct incoming wavefronts from Shack-Hartmann wavefront sensor (WFS) data
2. Atmospheric tomography: Reconstruct atmosphere from incoming wavefronts → Gradient-based method
3. Fitting step: Compute optimal mirror shape(s) from the reconstructed atmosphere



Input: Shack-Hartmann wavefront sensor (WFS) data of guide stars in directions α_g, $g = 1,\dots,G$

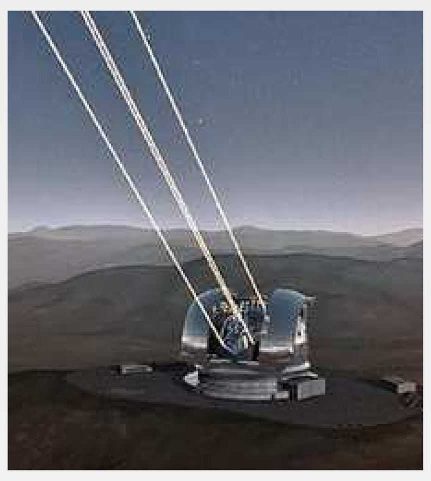
$$\Gamma : H^1(\Omega_D) \to \mathbb{R}^{2\#sub},$$
$$s^x_{\alpha_g} = \frac{1}{|\Omega_{ij}|}\int_{\Omega_{ij}} \frac{\partial \varphi_{\alpha_g}}{\partial x}\, d(x,y)\,,$$
$$s^y_{\alpha_g} = \frac{1}{|\Omega_{ij}|}\int_{\Omega_{ij}} \frac{\partial \varphi_{\alpha_g}}{\partial y}\, d(x,y)\,.$$

with the aperture Ω_D subdivided in sub-apertures Ω_{ij}.

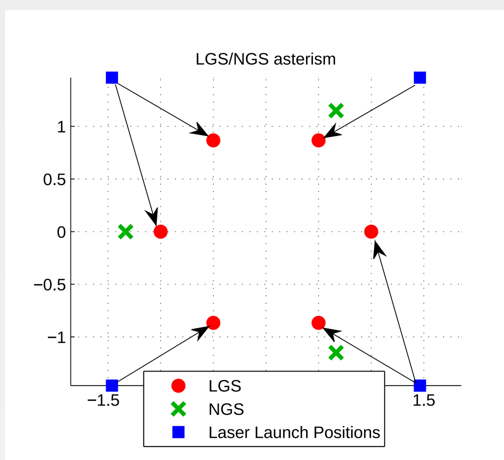
Output: DM shape

In the following, we propose a Gradient-based method for the atmospheric tomography. The main goal of this iterative approach is the comparability with the MVM in quality and a significantly lower computational cost.

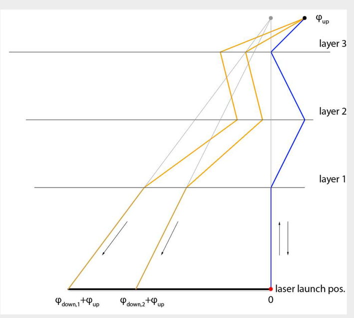
LGS deficiencies



With NGS only a low sky coverage can be reached, thus, artificial LGS are created with laser beacons. LGS suffer compared to NGS from cone effect, tip/tilt indetermination and spot elongation



Tip-tilt indetermination:



- Tip/tilt components are wrongly measured in a SH WFS
- remove wrong tip/tilt from incoming LGS wavefronts
- Tip/tilt removal operator: $\mathbf{\Pi} = (\underbrace{\Pi,\dots,\Pi}_{G\times},\underbrace{Id,\dots,Id}_{N\times})$
- $\Pi\varphi_{\alpha_g}(r) = \varphi_{\alpha_g}(r) - x\cdot t^x - y \cdot t^y$

Gradient tilt:

$$t^x = \frac{1}{|\Omega_D|}\int_{\Omega_D}\frac{\partial}{\partial x}\varphi_{\alpha_g}(x,y)d(x,y)$$
$$t^y = \frac{1}{|\Omega_D|}\int_{\Omega_D}\frac{\partial}{\partial y}\varphi_{\alpha_g}(x,y)d(x,y)$$

Zernike tilt:

$$t^x = \frac{1}{c|\Omega_D|}\int_{\Omega_D} x\cdot\varphi_{\alpha_g}(x,y)d(x,y)$$
$$t^y = \frac{1}{c|\Omega_D|}\int_{\Omega_D} y\cdot\varphi_{\alpha_g}(x,y)d(x,y)$$

Spot elongation:

SH WFS measurements from LGS are affected by spot elongation due to the non-zero thickness of the sodium layer. Depending on the laser launch position (the LGS height and the sodium layer) one can model the corresponding covariance matrix for the noise:

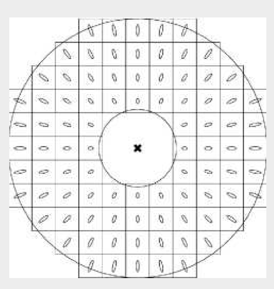
exact WFS data: $s_{\alpha_g} = [s^x_{\alpha_g}\, s^y_{\alpha_g}]$

noisy WFS data: $s^\delta_{\alpha_g} = s_{\alpha_g} + C^{1/2}_{\alpha_g}\eta$, with η white noise.

noisy wavefront: $\varphi^\delta_{\alpha_g} = \Gamma^\dagger(s_{\alpha_g} + C^{1/2}_{\alpha_g}\eta)$

covariance of noisy wavefront: $\mathrm{cov}(\varphi^\delta_{\alpha_g}) = \Gamma^\dagger C_{\alpha_g}(\Gamma^\dagger)^T$

$\mathrm{cov}(\boldsymbol{\varphi}) = \Gamma^\dagger C_\eta(\Gamma^\dagger)^T =: \overline{C_\eta}$ with $C_\eta = \mathrm{diag}(C_{\alpha_1},\dots,C_{\alpha_G})$.



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References

[1] R. Ramlau and M. Rosensteiner: *An efficient solution to the atmospheric turbulence tomography problem using Kaczmarz iteration.* Inverse Problems, 28 (2012)

[2] M. Rosensteiner: *Wavefront reconstruction for extremely large telescopes via CuRe with domain decomposition.* J. Opt. Soc. Am. A, 11 (2012)

[3] R. Ramlau, A. Obereder, M. Rosensteiner, D. Saxenhuber: *Efficient iterative tip/tilt reconstruction for atmospheric tomography.* Inverse Problems in Science and Engineering (2014)

[4] R. Ramlau, D. Saxenhuber, M. Yudytskiy: *Iterative reconstruction methods in atmospheric tomography: FEWHA, Kaczmarz and Gradient-based algorithm.* Proc. SPIE 9148, Adaptive Optics Systems IV (2014)

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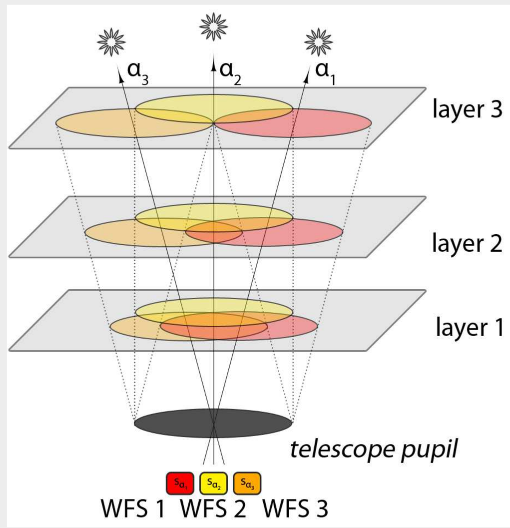
Atmospheric Tomography



Input:
- reconstructed incoming wavefronts φ_{α_g} from LGS $g = 1,\dots,G$ and NGS $n = 1,\dots,N$ on Ω_D (aperture)

Goal:
- fast reconstruction of discretized atmosphere $\mathbf{\Phi} = (\Phi^{(1)},\cdots,\Phi^{(L)})^T$ on $\mathbf{\Omega}_l$, $l = 1,\dots,L$

$\Longrightarrow$ **ill-posed inverse problem**, requires **regularization**.

Statistics of the atmosphere:
To model the atmosphere, a finite number of layers $l = 1,\dots,L$ is used. Each layer can be described e.g. by the van Karman or the Kolmogorov turbulence model with the covariance matrix $C^{(l)}_\phi$ and the c_n^2-profile γ_l measuring the turbulence strength of layer l.

Forward operator: using geometric light propagation [1]

$$\mathbf{A}_{\boldsymbol{\alpha}_g} : \bigotimes_{l=1}^{L} L_2(\mathbf{\Omega}_l) \to L_2(\mathbf{\Omega}_D)$$
$$\mathbf{A}_{\boldsymbol{\alpha}_g}\mathbf{\Phi} := \sum_{l=1}^{L}\Phi^{(l)}(c_l\mathbf{r} + h_l\boldsymbol{\alpha}_g)$$
$$\langle\mathbf{\Phi},\mathbf{\Psi}\rangle := \sum_{l=1}^{L}\frac{1}{\gamma_l}\langle\Phi^{(l)},\Psi^{(l)}\rangle_{L_2(\mathbf{\Omega}_l)}\,,$$

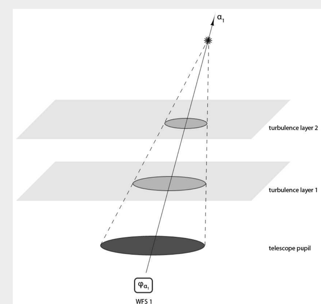
The L_2-adjoint:

$$\mathbf{A}^*_{\boldsymbol{\alpha}_g} : L_2(\mathbf{\Omega}_D) \to \bigotimes_{l=1}^{L} L_2(\mathbf{\Omega}_l)$$
$$\mathbf{A}^*_{\boldsymbol{\alpha}_g}\mathbf{\Psi} = \left[\gamma_l\mathbf{\Psi}\left(\frac{\mathbf{r}-\alpha_g h_l}{c_l}\right)\chi_{\Omega_D(h_l\alpha_g)}\left(\frac{1}{c_l}\mathbf{r}\right)\right]_{l=1}^{L}$$

Cone effect: introduce a scaling factor c_l

$$c_l := \begin{cases} 1\,, & \text{for NGS} \\ 1 - \frac{h_l}{h_{LGS}}, & \text{for LGS}\end{cases}$$

$h_{LGS}\,\dots$ LGS height, i.e. 90km



A Gradient-based method

$$\text{Solve } \mathbf{A\Phi} = \boldsymbol{\varphi} \iff \begin{pmatrix}\mathbf{A}_{\boldsymbol{\alpha}_1}\\ \vdots \\ \mathbf{A}_{\boldsymbol{\alpha}_{G+N}}\end{pmatrix}\mathbf{\Phi} = \begin{pmatrix}\varphi_{\alpha_1}\\ \vdots \\ \varphi_{\alpha_{G+N}}\end{pmatrix} = \boldsymbol{\varphi}$$

Approach without noise models:
Minimize the least-squares functional

$$J(\mathbf{\Phi}) = \|\mathbf{A\Phi} - \boldsymbol{\varphi}\|^2_{[L^2(\mathbf{\Omega}_D)]^{G+N}} \to \min\,,$$
$$J'(\mathbf{\Phi}) = -2\mathbf{A}^*\mathbf{\Pi}(\boldsymbol{\varphi} - \mathbf{A\Phi})$$

Approach with noise models:
Minimize the Tikhonov-type functional

$$J(\mathbf{\Phi}) = \|\mathbf{A\Phi} - \boldsymbol{\varphi}\|^2_{\overline{C_\eta}^{-1}} + \alpha_\Phi\|\mathbf{\Phi}\|^2_{C_\phi^{-1}} \to \min\,,$$
$$J'(\mathbf{\Phi}) = -2\mathbf{A}^*\mathbf{\Pi}\overline{C_\eta}^{-1}\mathbf{\Pi}(\boldsymbol{\varphi} - \mathbf{A\Phi}) + 2\alpha_\Phi C_\phi^{-1}\mathbf{\Phi}$$

Steepest descent iteration:

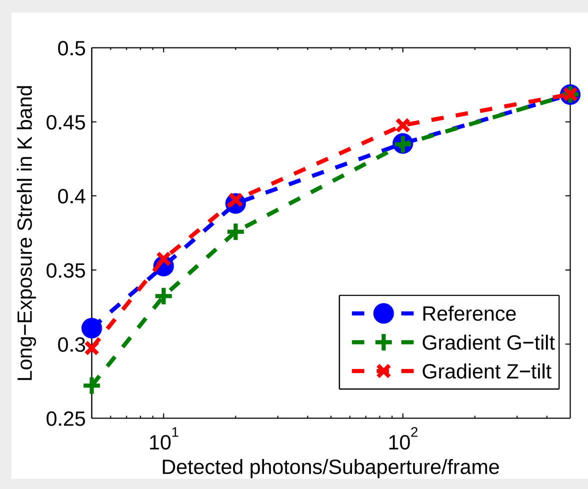
$$\mathbf{\Phi}_{i+1} = \mathbf{\Phi}_i - \tau_i J'(\mathbf{\Phi}_j)$$
$$\tau_i = \min_{t\in[0,\infty)} J(\mathbf{\Phi}_i - t\mathbf{d_i})$$

Algorithm 1 Gradient-based method without noise models

```
Choose Φ_0.
for i = 1, ... do
  Φ_i = Φ_{i-1},
  for g = 1, ..., G do
    residual_g = φ_{α_g} − A_{α_g} Φ_i
    gradient_g = (A_{α_g})* Π residual_g
    r_i = r_i + L· residual_g
    g_i = g_i + gradient_g
  end for
  for g = G+1, ..., G+N do
    residual_g = φ_{α_g} − A_{α_g} Φ_i
    gradient_g = (A_{α_g})* residual_g
    r_i = r_i + L· residual_g
    g_i = g_i + gradient_g
  end for
  stepsize = (g_i^T · g_i)/(r_i^T · r_i)
  Φ_i = Φ_i + stepsize · γ · g_i
end for
```

Simulation results

All results below were obtained for the E-ELT on the ESO end-to-end simulator, OCTOPUS.



CuReD	$20n$
Grad.method per it	$(16L-4)n$
Projection step	$7(L-1)n$

Computational cost per process

Multi Object Adaptive Optics (MOAO)
- 42m E-ELT, central obstruction
- nine 84×84 SH-WFS, 10 arcmin FoV
- 6 LGS (3.75 arcmin), 3 NGS (5 arcmin)
- 1 ground DM, open loop control
- varying photon noise, w/o spot elongation
- Reference: FrIM (e.g. Tallon et al. 2007)
- wavefront reconstruction: CuReD [2]
- Gradient w/o noise models
- 9 reconstruction layers (between 0-12km)
- projection through atmosphere onto DM